High Frequency Radiation Pattern Analysis for Antennas Mounted on Material-Coated Conducting Platforms of General Shape

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Abstract: Despite the ever-increasing availability of computational resources, high-frequency asymptotic techniques continue to play an important role in computational electromagnetics (CEM) analyses involving electrically large platforms. For purposes of antenna-pattern analysis, the Uniform Geometrical Theory of Diffraction (UTD) may be considered to be a highly advantageous technique because it includes creeping-wave field components that are essential to accurate pattern computation in the shadow region. While the implementation of the UTD ha historically been limited to platform models derived from simple canonical shapes such as cylinders, plates and ellipsoids, implementation of the UTD for a very general class of platform models represented in terms of triangular facets has recently been demonstrated. Another significant limitation of the UTD has been the restriction to the case of perfect conductivity, which has been required due to the lack of available solutions for other cases. In particular, available creeping-ray solutions for fields in the shadow region of a convex body have been limited to the case of perfect conductivity. Recently, an asymptotic approximate Maxwell solution for boundary layer fields excited by an arbitrarily-oriented tangential magnetic source residing on a convex impedance surface of general shape was obtained in creeping-ray modal format. In this paper, a companion solution for fields radiated into the far zone of a convex singly-coated surface of general shape is developed. An outline for a UTD treatment of pattern analysis for antennas mounted on singly-coated platforms of general shape is indicated.

Keywords: radiation, convex surface, coating, asymptotic techniques, antenna patterns

1. Introduction

The construction of an asymptotic approximate Maxwell solution for boundary-layer fields excited by an arbitrarily-oriented tangential magnetic source residing on an arbitrary convex impedance cylinder has been recently described in a pair of papers [1],[2]. First, the application of a series of transformations and substitutions to the canonical solution for fields excited by an axial surface magnetic dipole on an impedance circular cylinder was demonstrated in [1] to lead to a set of field components (**E**,**H**) that act as an order $k^{-2/3}$ solution for both the boundary conditions and the Maxwell Equations written in coordinates appropriate to the boundary layer of an arbitrary convex surface. Subsequently, in [2], an alternative approximate Maxwell solution in general coordinates was obtained from the canonical solution for the case of azimuthal dipole excitation, and was combined with the solution from [1] to provide a dyadic Green's-function representation for the boundary-layer fields excited by a tangential magnetic source on a convex impedance surface. Crucial to the construction of the alternative solution in [2] was the identification of a simple relationship between the azimuthal-source solution and the axial-source solution under duality transformation. In [3], an approximate solution for radiated fields in the far shadow zone of a convex singly-coated conductor was obtained applying a transformation/substitution process (similar to that in [1]) to the canonical solution for fields excited in the far shadow zone by an axial magnetic source embedded in a material layer surrounding a conducting circular cylinder. For the case of a tangential magnetic source residing on the exterior surface of a singly-coated conducting circular cylinder, a relationship between azimuthal-source and axial-source canonical solutions may

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be identified. While this relationship is somewhat more complicated than that described in [2], it can nonetheless be employed to construct an alternative solution to [3] for the case of a surface tangential source. Combination of this solution with the solution in [3] then leads to a dyadic Green's function representation of fields radiated into the far shadow zone of a singly-coated convex conducting surface.

2. Adaptation of the Axial-Source Canonical Solution

In [3], the construction of radiated field components that approximately satisfy the Maxwell Equations in the shadow zone of an arbitrary convex singly-coated conductor begins with the canonical solution for fields radiated by an axial magnetic dipole embedded in a material layer surrounding a conducting circular cylinder (see, e.g. [4]). The starting point for the transformation/substitution process is the canonical solution represented in an asymptotic creeping-ray modal format as obtained from the exact spectral-integral representation via a series of familiar steps. These steps include residue-series evaluation of azimuthal-wavenumber integrals, stationary-phase evaluation of axial-wavenumber integrals, and approximation of Hankel functions via Airy functions or via the Debye approximation as appropriate to the argument size. The asymptotic field components (**E**,**H**) thereby obtained are then transformed from the native (ρ,ϕ,z) cylindrical coordinate system to a ray-based system (s,θ_c,t) specific to the cylindrical geometry defined with respect to the coating outer radius, b, as follows:

$$z - z' = (s+t)\cos\theta_C \tag{1a}$$

$$\rho = \sqrt{b^2 + s^2 \sin^2\theta_c} \tag{1b}$$

And

$$\varphi - \varphi' = \frac{t}{b}\sin\theta_c + \tan^{-1}(s\sin\theta_c / b) \tag{1c}$$

The asymptotic solution in cylinder-specific ray coordinates is then used as a template for the construction of an approximate solution to the Maxwell Equations written in ray coordinates (s,t,θ) appropriate for an arbitrary convex surface. Here, the pair (t,θ) identifies the point of ray tangency in geodesic polar coordinates centered at the source locus on an arbitrary surface, while s is the spatial ray length from the point of tangency to the field point.

The substitutions that are employed may be separated into two classes. The first class consists of substitutions which are motivated by a comparison between the metrics and curl operators appropriate to the (s,θ_c,t) and to the (s,t,θ) systems. Within the asymptotic cylinder solution $(\mathbf{E}_c,\mathbf{H}_c)$ written in cylinder-ray coordinates there occur a variety of factors whose appearance may be understood as deriving from their presence within the metric and curl for the (s,θ_c,t) system, together with the requirement that the field components satisfy the Maxwell Curl Equations (to some degree of approximation) in the (s,θ_c,t) system. Such factors are readily identifiable, and are replaced by corresponding factors associated with the (s,t,θ) system. In particular, the following substitutions are enforced at the point of tangency:

$$\sqrt{1+\frac{s^2}{b^2}\sin^4\theta_c} \;\Rightarrow\; \sqrt{1+\frac{s^2}{{\rho_g}^2}} \tag{2a}$$

and

$$\sqrt{(s+t)^2+\frac{s^2t^2}{b^2}\sin^2\theta_c\cos^2\theta_c} \Rightarrow \sqrt{(\rho^2+s)^2+s^2T^2(\rho^d)^2} \tag{2b}$$

where ρ^d is the surface-wavefront radius of curvature and where with T represents the torsion and ρ_g represents the radius of curvature of the geodesic ray path. A further substitution that falls into the first class is the identification of the surface-ray length, t, as it appears in certain instances within the cylinder solution, with ρ^d,. Finally, "wave-spreading" factors that are specific to the cylindrical geometry are replaced by corresponding factors suitable for arbitrary convex geometry. [4].

The above substitutions are of a purely geometrical nature. Substitutions that fall within the second class are based on a more physical idea, namely that the to low order the propagation of the creeping ray should occur independently of the surface radius of curvature transverse to the direction of propagation. This physical idea is implemented via the replacement of geometrical angle θ_c with angle γ defined in terms of the UTD generalized torsion factor [5], such that

$$\sin\gamma = \frac{1}{\sqrt{1+(T\rho_g)^2}} \tag{3a}$$

$$\cos\gamma = \frac{T\rho_g}{\sqrt{1+(T\rho_g)^2}} \tag{3b}$$

as well as via certain associated substitutions, such as the replacement of the cylinder radius, b, by:

$$b \Rightarrow \frac{\rho_g}{1+(T\rho_g)^2} \tag{4}$$

In particular, a representation for the exponential factors governing the attenuation of the creeping-ray modes is obtained from substitutions falling into the second class. Whereas in the cylinder case the attenuation of mode p is governed by the Fock parameter, ξ, and by a root, τ_p, that can be identified with a pole location encountered in the lower half-plane in a residue-series evaluation of integrals over azimuthal wavenumber,, in the case of arbitrary cionvex geometry the attenuation of mode p is taken to be governed by a generalized factor Φ_p, i.e.

$$e^{-j\xi\tau_p} \Rightarrow e^{-j\Phi_p(t,\theta)} \tag{5}$$

The generalized factor Φ_p is given by

$$\Phi_p(t,\theta) = \int_{Q'}^{Q} \frac{m_g(t',\theta)\tau_p(t',\theta)}{\rho_g(t',\theta)}dt' \tag{6}$$

where $m_g(t',\theta) = [k\rho_g(t',\theta)]^{1/3}$, with k representing the wavenumber, and where at any point (t',θ) along the surface geodesic, the root, $\tau_p(t',\theta)$ is identified with a root that appears in the canonical-cylinder solution for a

cylinder whose radius is defined according to (4), and for a geodesic direction identified with angle $\gamma(t',\theta)$. Lastly, Q' is the source locus, while Q is the point of tangency.

Introduction of the above substitutions into the leading-order terms in the canonical coated-cylinder asymptotic solution for axial magnetic-dipole excitation leads to a trial solution expressed in coordinates that are independent of the cylindrical geometry. Terms at next-to-leading-order can be introduced as unknowns. Application of the Maxwell Equations in the (s,t,θ) system leads to a set of restrictions imposed on the unknowns. While the restrictions imposed upon the unknowns are consistent, indicating that the Maxwell Equations are satisfied through next-to-leading order; the unknowns are not uniquely defined. Only the leading-order terms are thereby available. The associated field components will be denoted as $(\mathbf{E_z},\mathbf{H_z})$ to indicate that they have been obtained by modification to a canonical solution for an axial or z-directed source.

3. Adaptation of the Azimuthal-Source Canonical Solution

Given the approximate Maxwell solution $(\mathbf{E}_z,\mathbf{H}_z)$ for creeping-ray fields radiated into the far zone of a singly-coated surface of arbitrary curvature obtained above as a generalization of the canonical cylinder solution with axial-source excitation, it is possible to obtain an alternative solution corresponding to an azimuthal or phi-directed source in a simple way. In the exact spectral-integral representation for the axial-source solution, it is useful to define the functions $C_s(\nu,\alpha)$ and $C_h(\nu,\alpha)$ as

$$C_s(\tau_\rho) = -j\frac{k_c^2\mu_o}{\beta_c k_o \mu_c}\frac{H_\nu^{(1)\prime}(\beta_c b)H_\nu^{(2)}(\beta_c a) - H_\nu^{(2)\prime}(\beta_c b)H_\nu^{(1)}(\beta_c a)}{H_\nu^{(1)}(\beta_c b)H_\nu^{(2)}(\beta_c a) - H_\nu^{(2)}(\beta_c b)H_\nu^{(1)}(\beta_c a)} \tag{7a}$$

$$C_h(\tau_\rho) = -j\frac{k_c^2\varepsilon_o}{\beta_c k_o \varepsilon_c}\frac{H_\nu^{(1)\prime}(\beta_c b)H_\nu^{(2)\prime}(\beta_c a) - H_\nu^{(2)\prime}(\beta_c b)H_\nu^{(1)\prime}(\beta_c a)}{H_\nu^{(1)}(\beta_c b)H_\nu^{(2)\prime}(\beta_c a) - H_\nu^{(2)}(\beta_c b)H_\nu^{(1)\prime}(\beta_c a)} \tag{7b}$$

where k_c is the wavenumber inside the coating, a is the interior radius of the coating, μ_0 is the permeability of free space, μ_c is the premeability inside the coating, ε_o is the permittivity of free space, ε_c is the permittivity inside the coating, $\beta_c = (k_c^2-\alpha^2)^{1/2}$, and where $H_\nu^{(1,2)}$ are, respectively, Hankel functions of the first and second kind, of order ν. Let $g(\nu,\alpha,C_h,C_s)$ and $f(\nu,\alpha,C_h,C_s)$ represent spectral functions that have an explicit dependence on C_h and C_s, and let F be given by

$$F = \iint f(\nu,\alpha,C_h,C_s)\,d\nu d\alpha \tag{8}$$

Let the operators Δ and Δ' be defined so that

$$g\Delta F \equiv \iint g(\nu,\alpha,C_h,C_s)f(\nu,\alpha,C_h,C_s)d\nu d\alpha \tag{9a}$$

$$g\Delta' F \equiv \iint g(\nu,\alpha,C_h,C_s)f(\nu,\alpha,C_s,C_h)d\nu d\alpha \tag{9b}$$

With the aid of these definitions, the field components excited by an infinitesinal azimuthal magnetic dipole source on a coated circular cylinder may be related to the field components excited by an axial source of equivalent strength via

$$\mathbf{E}_p{}^c = -\eta_o C_s \Delta' \mathbf{H}_z{}^c - \frac{\nu\alpha}{\beta_c{}^z b} \Delta \mathbf{E}_z{}^c \tag{10a}$$

$$\mathbf{H}_p{}^c = \frac{1}{\eta_o} C_s \Delta' \mathbf{E}_z{}^c - \frac{\nu\alpha}{\beta_c{}^z b} \Delta \mathbf{H}_z{}^c \tag{10b}$$

If the Cauchy theorem is applied to the azimuthal wavenumber integrals in (10a) and (10b), and if the axial wavenumber integrals are treated via stationary phase, it is then apparent that for each creeping-ray mode, p, the mode-p contribution to the azimuthal-source fields consists of two pieces, where the first piece may be identified with the axial-source mode-p contribution under duality-transformation along with the transformation $C_h \Leftrightarrow C_s$, and multiplied by C_s, while the second piece may be identified with the axial-source mode-p contribution multiplied by $\nu_p k\cos\theta_c/\beta_c{}^2 b$. Because of these relations, an approximate Maxwell solution for the azimuthal-source case is available from the axial-source solution by applying a simple transformation. For mode p, the components of the alternative solution are given by

$$\mathbf{E}_n(\tau_p, C_h, C_s) = -\eta_o C_s(\tau_p) \mathbf{H}_s(\tau_p, C_s, C_h) - \frac{\upsilon_p k\cos\gamma}{\beta_c{}^2 b} \mathbf{E}_s(\tau_p, C_h, C_s) \tag{11a}$$

$$\mathbf{H}_n(\tau_p, C_h, C_s) = \frac{1}{\eta_o} C_s(\tau_p) \mathbf{E}_s(\tau_p, C_s, C_h) - \frac{\upsilon_p k\cos\gamma}{\beta_c{}^2 b} \mathbf{H}_s(\tau_p, C_h, C_s) \tag{11b}$$

where ν_p, C_h, and C_s are to be evaluated at the point of tangency and where subscript "n" denotes the "new" solution and subscript s denotes the reference [3] ("San Antonio") solution. Note that the requirement that the functions C_h, and C_s have slowly varying dependence on the field point coordinates, which is explicit here, was implicit in the development of the axial-source solution.

4. Dyadic Green's Function

Let $\mathbf{F}_z$ represent solution pair $(\mathbf{E}_z,\mathbf{H}_z)$ developed from the axial-dipole cylinder solution in [3], and let $\mathbf{F}_p$ represent the solution pair $(\mathbf{E}_p,\mathbf{H}_p)$ developed from the azimuthal dipole cylinder solution as above.. If $\mathbf{F}$ represents the creepint-ray fields $(\mathbf{E},\mathbf{H})$ radiated into the far zone of an arbitrary convex surface by an arbitrarily-directed infinitesimal magnetic dipole source then $\mathbf{F}$ can be expressed as

$$\mathbf{F} = \{ -\hat{b}'\bullet\hat{p}_m(\sin\gamma''\mathbf{F}_z - \cos\gamma''\mathbf{F}_p) + \hat{t}'\bullet\hat{p}_m(\cos\gamma''\mathbf{F}_z + \sin\gamma''\mathbf{F}_p)\}\sin\gamma \tag{12}$$

where $\hat{b}'$ and $\hat{t}'$ are unit vectors respectively binormal and tangential to the geodesic at the source locus, $\hat{p}_m$ represents a unit vector parallel to the source orientation, and the prime applied to γ means that evaluation occurs at the source locus... Upon restriction to the case of spherical geometry, (12) reduces to the leading-order terms in asymptotic version of the canonical coated-sphere solution provided by Munk [4].

5. Conclusion

An approximate Maxwell solution for shadow-region fields radiated into the far zone of a singly-coated conducting surface of arbitrary curvature by an arbitrarily-oriented surface tangential magnetic dipole source has been obtained. While it has been demonstrated that the Maxwell Equations can be satisfied through order $k^{-2/3}$, the next-to-leading-order terms are not uniquely defined, so that only the leading-order terms are available. Combination with other solutions will be necessary to construct a complete uniform GTD treatment for radiation from coated surfaces of general curvature.

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